

A User's Guide to Algebraic Topology

by

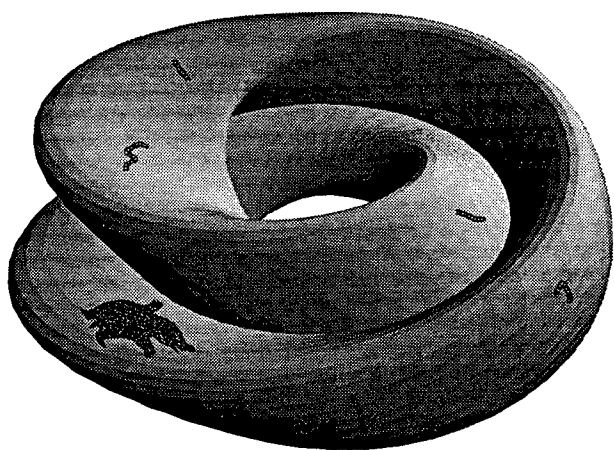
C. T. J. Dodson

*University of Toronto,
Ontario, Canada*

and

Phillip E. Parker

*Wichita State University,
Kansas, U.S.A.*



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